



Two-Scale continuum damage mechanics model with separate deviatoric and volumetric damage for multiaxial fatigue life prediction

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ABSTRACT

Multiaxial fatigue life prediction remains a central challenge in structural integrity assessment. Conventional continuum damage mechanics (CDM) formulations based on a single scalar damage variable cannot distinguish between deviatoric (shear-driven) and volumetric (void-growth-driven) damage mechanisms. A two-scale CDM model coupling macroscopic elasticity with microscopic elastoplasticity is developed, in which the scalar damage variable is replaced by separate deviatoric and volumetric damage variables governing shear and bulk stiffness degradation independently. Evolution is driven by stress-state-dependent damage strength functions, enabling unified fatigue-life prediction and mechanism-resolved outputs including damage trajectories and volumetric damage fraction. The model is calibrated against 41 constant-amplitude fatigue tests of 7050-T7451 aluminum alloy under three loading paths, and a controlled ablation study confirms that each constitutive innovation contributes independently to the overall accuracy. The predicted damage partitioning is consistent with the expected failure mechanisms for each path, with zero volumetric damage in pure torsion confirming the structural correctness of the separation. Independent validation using MIL-HDBK-5H S-N data without parameter refitting demonstrates robust parameter transferability. Application to an aircraft cargo-door lifting mechanism under a full multiaxial stress history yields a life prediction with pronounced volumetric-damage dominance at failure, and benchmarking against three established engineering fatigue methods shows comparable life estimates while additionally providing damage evolution histories and failure-mechanism partitioning. Overall, the proposed framework offers a physical basis for simultaneous fatigue-life prediction, failure-mode interpretation, and engineering safety assessment under multiaxial loading.

multiaxial fatigue assessment.

1. Introduction

Multiaxial fatigue life prediction remains a central problem in structural integrity assessment for engineering components subjected to complex loading histories. In practical structures, fatigue-critical locations often experience a full multiaxial stress-component history under varying stress states, and predictive performance depends not only on life-estimation accuracy but also on whether the underlying damage mechanisms are represented consistently across loading paths. In particular, the relative contributions of shear-driven (deviatoric) damage and void-growth-driven (volumetric) damage may vary significantly with stress state, posing a persistent challenge to fatigue models based on a single scalar damage measure. A physically interpretable framework that can simultaneously provide fatigue-life prediction and mechanism-resolved damage evolution is therefore highly desirable for

1.1. Literature review

Existing multiaxial fatigue-life prediction methods are generally developed along two principal routes. One route is based on critical-plane concepts or equivalent parameters, in which a governing damage measure is extracted from multiaxial stress/strain paths. Gao et al. [1] extended the critical-plane concept to multiaxial random loading. The other route is based on stress invariants, where combinations of hydrostatic and shear components are used to characterize stress-state effects. In contrast to parameter-based approaches, damage mechanics represents material degradation through internal variables, including GTN-type micromechanical models and continuum damage mechanics (CDM) formulations; Piska et al. [2] provided a recent overview of computational damage frameworks. In ductile damage modelling, Wu

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Nomenclature			
a	Eshelby–Kröner localization constant	S_s, S_v	deviatoric and volumetric damage strength functions
b	Armstrong–Frederick saturation parameter	$S_s, 0.33, S_s, 0.0$	deviatoric damage strength at reference tension and shear states
$\mathbb{C}$	fourth-order elastic stiffness tensor	$S_v, 0.33, S_v, 0.0$	volumetric damage strength at reference tension and shear states
D_s, D_v	deviatoric and volumetric damage variables	u	microscopic Helmholtz free-energy density
$D_{s,c} D_{v,c} D_c$	critical values of deviatoric damage, volumetric damage, and combined damage	x	unknown vector in the coupled plasticity–damage update
D_{ef}	normalized effective damage index	Y_s, Y_v	thermodynamic driving forces (energy release rates) associated with D_s and D_v
E	Young's modulus	$\alpha\mu$	microscopic backstress tensor
G, K	shear and bulk moduli (undamaged)	$\dot{\gamma}\mu, \Delta\gamma$	microscopic plastic multiplier rate; plastic multiplier increment
G_c, K_c	current shear and bulk moduli	$\Delta D_s, \Delta D_v$	deviatoric and volumetric damage increments
H_k	kinematic hardening modulus	$\varepsilon, \varepsilon e$	macroscopic total and elastic strain tensors
$\mathbf{I}, \mathbf{I}_s$	second- and fourth-order identity tensors	$\varepsilon\mu, \varepsilon\mu e, \varepsilon\mu p$	microscopic total, elastic, and plastic strains
$\mathbf{J}$	Jacobian matrix of the residual system	$\eta\mu$	microscopic stress triaxiality
M	increments per loading cycle	θ	calibration parameter vector ($S_s, 0.33, S_s, 0.0, S_v, 0.33$)
$\mathbf{n}$	radial-return flow direction	$\nu, \nu c$	Poisson's ratio (undamaged); current Poisson's ratio
N_f	predicted fatigue life	$\xi\mu$	normalized third invariant of the microscopic deviatoric stress tensor
$\mathbf{P}_v, \mathbf{P}_D$	volumetric and deviatoric projectors	$\sigma\mu$	microscopic Cauchy stress tensor
p^μ	microscopic hydrostatic pressure	$\sigma_\infty f$	microscopic fatigue limit
$q^{-\mu}$	effective Mises stress (from $\mathbf{r}^\mu$)	σF	fracture stress
q^μ	von Mises equivalent stress (from $\mathbf{S}^\mu$)	σu	ultimate tensile strength
$\mathbf{r}^\mu$	effective deviatoric stress tensor	$\varphi\mu$	microscopic yield function
$\mathbf{R}$	Newton–Raphson residual vector	Φ	cycle-end failure indicator
s	damage exponent		
S_μ	microscopic deviatoric stress tensor		

et al. [3] described void–shear competition through a dual-variable GTN-type formulation, whereas He et al. [4] introduced stress-state modulation to separate competing mechanisms, thereby highlighting the importance of mechanism-sensitive modelling. In fatigue damage modelling, Araghi et al. [5] showed that CDM predictions are sensitive to proportional and non-proportional loading paths, indicating the need for robust cross-path predictive capability. Gonçalves Junior et al. [6] reproduced a rapid high-cycle fatigue test of high Mn-TWIP steel using a CDM-based constitutive model and finite-element implementation, and improved computational efficiency through a cycle-jump strategy.

Notwithstanding these developments, two issues remain central in multiaxial fatigue CDM, namely applicability in force-controlled high-cycle fatigue and robust stress-state dependence across loading paths. Standard CDM formulations often face an applicability limit in force-controlled high-cycle fatigue, where the macroscopic response remains nearly elastic while failure still develops. Fang et al. [7] showed that two-scale coupling can alleviate this limitation by linking macroscopic near-elastic response to microscopic dissipation. Robust stress-state dependence is likewise essential for cross-path performance. Malcher and Mamiya [8] modulated damage strength using stress triaxiality and a normalized third invariant to reduce sensitivity to calibration conditions. Within a CDM–XFEM framework for fatigue crack growth, Pandey et al. [9] further showed that the treatment of stress triaxiality is critical for capturing crack-tip constraint effects across specimen configurations. Carol et al. [10] established the theoretical basis for volumetric/deviatoric decomposition, and Brüning et al. [11] extended stress-state parameterization to plastically anisotropic metals using generalized triaxiality and Lode parameters. Han et al. [12] developed a gradient damage model for ductile fracture with implementation and experimental validation, further enriching mechanism-sensitive damage modelling for fracture analysis. Bonora and Testa [13] incorporated stress triaxiality and shear-controlled mechanisms into a plasticity–damage framework and demonstrated predictive capability under different stress states. Taken together, these studies provide the theoretical and mechanistic basis for stress-state-sensitive modelling and volumetric/deviatoric effect separation, while a multiaxial fatigue framework that simultaneously unifies

damage-evolution tracking, mechanism-resolved diagnosis, and engineering applicability has not yet been established.

Compared with empirical parameter-based methods that provide only a scalar life estimate, CDM can provide both fatigue-life prediction and damage-evolution trajectories, thereby supporting mechanism-resolved diagnosis and failure-mode interpretation. For engineering-scale applications, He et al. [14] incorporated residual-stress effects into a continuum damage model and implemented high-cycle fatigue assessment through a UMAT with a load-block strategy. Han et al. [15] further incorporated wear effects and cyclic plasticity accumulation into degradation modelling under multi-source uncertainty for aircraft mechanisms. Ye et al. [16] combined macroscopic CDM characterization with micromechanical void-based interpretation, supporting the coexistence of volumetric void-growth and shear-driven damage. Sadeghi Nezhad et al. [17] reported a VUMAT implementation and a parameter-identification workflow for a Lemaitre-type model, improving the reproducibility of CDM-based engineering applications. Parameter identification nevertheless remains a critical challenge because of parameter correlation and identifiability. Adeli et al. [18] showed that posterior convergence is sensitive to the updating strategy, and Saji et al. [19] proposed a multi-stage calibration framework to improve robustness and computational efficiency in constitutive parameter estimation. Liu et al. [20] coupled a CDM framework with an error-trained BP-ANN correction strategy for HCF life prediction of aluminum alloys, illustrating a hybrid route to improve predictive accuracy in engineering-oriented fatigue assessment. Han et al. [21] extended this line of research to a two-scale CDM–Bayesian framework for aircraft mechanisms, integrating parameter identification, external validation, and hotspot migration. Li et al. [22] further revised and validated the damage–plastic-strain coupling assumption in a Lemaitre-type model, emphasizing that evolution-law refinements should remain consistent with observable degradation measures. For engineering extrapolation and failure-assessment robustness, Fu et al. [23] developed a reproducible CDM-based prediction framework accounting for mean-stress and anisotropy effects, while Zhang et al. [24] extended CDM-based fatigue-life prediction to defect-dominated degradation conditions.

Wang et al. [25] incorporated defect-geometry effects into a CDM-based fatigue framework for welded joints, and Xu et al. [26] combined CDM with fracture-mechanics-based formulations for fretting-fatigue life prediction, demonstrating the adaptability of damage-based frameworks to defect-sensitive and mechanism-coupled fatigue problems. Hottin et al. [27] further extended CDM-based multiaxial fatigue-life prediction to rubber, suggesting transferability of the modelling framework across material systems. Complementary developments such as CDM–XFEM coupling [28], anisotropic damage formulations [29], and regularized CDM models for fatigue crack growth [30] further reflect ongoing efforts to improve the robustness and transferability of damage-based assessment.

1.2. Current challenges

In summary, multiaxial fatigue life prediction still faces four key challenges in engineering applications:

- (1) Conventional scalar-damage CDM formulations cannot distinguish deviatoric and volumetric damage mechanisms. Even with stress-state-dependent damage strength, a single damage variable cannot separately track shear-driven and void-growth-driven contributions under mixed-mode loading within a unified parameter set.
- (2) The contribution of each constitutive innovation within a given modelling framework is seldom isolated and quantified in a controlled manner. Most existing comparisons involve different code bases with different numerical settings, making it difficult to attribute accuracy differences to the constitutive formulation itself.
- (3) Model performance is often assessed only on calibration datasets, and independent external validation across loading paths and data sources remains limited. This restricts confidence in parameter transferability to conditions beyond the calibration domain.
- (4) Many existing engineering approaches, including both empirical critical-plane criteria and scalar CDM models, provide only fatigue-life estimates, with limited quantitative information for mechanism-resolved diagnosis under complex multiaxial loading.

1.3. Contributions

To address these challenges, the present study develops a two-scale CDM model coupling macroscopic elasticity with microscopic elastoplasticity. The main capabilities added by the present work are summarized as follows:

- (1) Separate deviatoric and volumetric damage variables, D_s and D_v , are introduced with stress-state-dependent damage strength functions $S_s(\eta^\mu, \xi^\mu)$ and $S_v(\eta^\mu, \xi^\mu)$, enabling each damage mechanism to evolve independently and allowing unified fatigue-life prediction across axial, torsional, and proportional loading paths.
- (2) A controlled ablation study is performed within the same numerical framework by progressively deactivating the proposed constitutive features, so that the respective contributions of stress-state-dependent damage strength and deviatoric/volumetric separation can be isolated and quantified.
- (3) A unified parameter set is calibrated from multiaxial fatigue data, externally validated on an independent handbook S–N dataset without parameter refitting, and further applied to an engineering component under a full multiaxial stress history, forming a three-level validation strategy that extends beyond calibration-only assessment.
- (4) Beyond fatigue-life estimation, the model outputs complete damage trajectories $D_s(N)$ and $D_v(N)$ together with mechanism-

diagnostic indicators, establishing a quantitative link between stress state and the dominant failure mechanism that is not available from single-scalar formulations.

1.4. Organization

The remainder of this paper is organized as follows. Section 2 presents the two-scale CDM formulation with separate deviatoric and volumetric damage variables and stress-state-dependent damage strength functions. Section 3 describes the parameter calibration, accuracy assessment with a controlled ablation study, and mechanism-resolved damage characterization. Then, in Section 4, engineering fatigue assessment and benchmarking against representative methods are carried out for an aircraft cargo-door lifting mechanism. Finally, Section 5 concludes the paper and outlines future work.

2. Theory and methods

2.1. Two-scale framework for fatigue life prediction

Under force-controlled high-cycle fatigue (HCF), the structural response remains nominally elastic at the macroscale, while cyclic plasticity may still develop locally at the microscale. To represent this scale separation, the present study adopts the two-scale framework of Araújo et al. [31], in which the macroscale supplies the cyclic loading input and the microscale governs the irreversible plastic evolution and associated damage accumulation responsible for fatigue life.

Under the small-strain assumption, the macroscopic response is taken to be purely elastic throughout the life analysis,

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e \quad (1)$$

where $\boldsymbol{\varepsilon}$ is the macroscopic total strain and $\boldsymbol{\varepsilon}^e$ is the macroscopic elastic strain.

At the microscopic scale, the strain is decomposed into elastic and plastic parts according to.

$$\boldsymbol{\varepsilon}^\mu = \boldsymbol{\varepsilon}^{\mu e} + \boldsymbol{\varepsilon}^{\mu p} \quad (2)$$

where $\boldsymbol{\varepsilon}^\mu$, $\boldsymbol{\varepsilon}^{\mu e}$, and $\boldsymbol{\varepsilon}^{\mu p}$ denote the microscopic total, elastic, and plastic strains, respectively.

To establish the coupling between the two scales, a strain-localization relation is adopted. This relation originates from the classical inclusion-based localization framework developed by Eshelby [32] and subsequently extended by Kröner [33]. In its general form, the localization relation expresses the microscopic total strain as

$$\boldsymbol{\varepsilon}^\mu = \boldsymbol{\varepsilon} + a(\boldsymbol{\varepsilon}^{\mu p} - \boldsymbol{\varepsilon}^p) \quad (3)$$

where $\boldsymbol{\varepsilon}^p$ denotes the macroscopic plastic strain. For the force-controlled HCF conditions considered in the present work, the macroscopic plastic strain is assumed to be zero or negligible, and the above localization relation therefore reduces to the form.

$$\boldsymbol{\varepsilon}^\mu = \boldsymbol{\varepsilon} + a\boldsymbol{\varepsilon}^{\mu p} \quad (4)$$

in which a is the localization constant. For the isotropic spherical-inclusion setting, a depends only on Poisson's ratio ν ,

$$a = \frac{2}{15} \frac{4 - 5\nu}{1 - \nu} \quad (5)$$

Substitution of Eq. (2) into Eq. (4) gives the microscopic elastic strain in terms of the macroscopic strain and the microscopic plastic-strain history,

$$\boldsymbol{\varepsilon}^{\mu e} = \boldsymbol{\varepsilon} + (a - 1)\boldsymbol{\varepsilon}^{\mu p} \quad (6)$$

Eq. (6) thus provides the constitutive connection between the two scales. The overall structure of the two-scale framework is illustrated in Fig. 1.

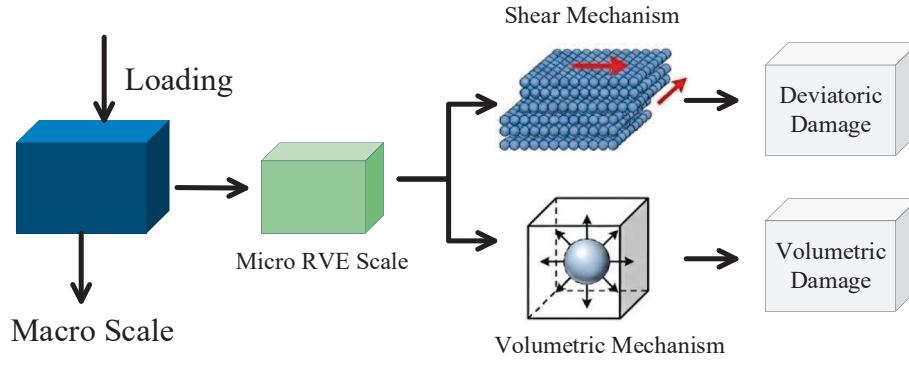


Fig. 1. Schematic of the two-scale damage modelling framework.

2.2. Microscopic constitutive formulation

Based on the scale transformation established in Section 2.1, the microscopic constitutive model is formulated in this section. The formulation combines volumetric/deviatoric degradation of microscopic elasticity, effective-stress-based microscopic plasticity, and the thermodynamic description of damage evolution and fatigue failure.

2.2.1. Volumetric/Deviatoric degradation of microscopic elasticity

In the classical single-scalar damage model, all elastic moduli degrade proportionally, which does not allow distinguishing between deviatoric and volumetric damage modes.

Under multiaxial fatigue loading, different stress states may activate different dominant microdamage mechanisms. To distinguish shear-dominated and void-related degradation within a continuum framework, the present study adopts the volumetric/deviatoric damage decomposition proposed by Carol et al. [10]. This split is introduced at the constitutive level as an energy-equivalent representation of the distinct contributions of the two mechanisms to stiffness degradation, rather than as a direct identification of damage with strain components. For an initially isotropic material, the elastic response can be conveniently expressed by means of the volumetric and deviatoric projection operators.

$$\mathbf{P}_V = \frac{1}{3}\mathbf{I} \otimes \mathbf{I}, \mathbf{P}_D = \mathbf{I}_s - \mathbf{P}_V \quad (7)$$

where $\mathbf{I}$ is the second-order identity tensor and $\mathbf{I}_s$ is the fourth-order symmetric identity tensor. The isotropic elastic stiffness tensor is then written in volumetric/deviatoric spectral form as

$$\mathbb{C} = 3K\mathbf{P}_V + 2G\mathbf{P}_D \quad (8)$$

where K and G denote the bulk and shear moduli of the reference isotropic material. This decomposition provides the constitutive basis for introducing independent degradation in the volumetric and deviatoric channels.

Accordingly, the present microscopic model introduces a volumetric damage variable D_v and a deviatoric damage variable D_s , which act independently on the bulk-related and shear-related elastic responses. Consistent with the extended isotropic degradation model of Carol et al. [10], the current volumetric and deviatoric constitutive contributions are allowed to evolve separately, so that the degradation of the two elastic channels is governed by different damage variables. The microscopic elastic response is therefore represented through its deviatoric and volumetric stress components. Denoting by

$$\mathbf{e}_D^{\mu e} = \mathbf{P}_D : \mathbf{e}^{\mu e} \quad (9)$$

the deviatoric part of the microscopic elastic strain tensor and by

$$\mathbf{e}_V^{\mu e} = \frac{1}{3}\text{tr}(\mathbf{e}^{\mu e}) \quad (10)$$

its volumetric component, the microscopic deviatoric stress and hydrostatic pressure are written as

$$\mathbf{S}^{\mu} = 2(1 - D_s)G\mathbf{e}_D^{\mu e} \quad (11)$$

$$p^{\mu} = 3(1 - D_v)K\mathbf{e}_V^{\mu e} \quad (12)$$

respectively, and the microscopic Cauchy stress is expressed as

$$\boldsymbol{\sigma}^{\mu} = \mathbf{S}^{\mu} + p^{\mu}\mathbf{I} \quad (13)$$

where $\mathbf{S}^{\mu}$ is the microscopic deviatoric stress tensor and p^{μ} is the microscopic hydrostatic pressure. These relations preserve the constitutive separation introduced in Eq. (8): D_s governs the degradation of the shear-modulus-related response, whereas D_v governs the degradation of the bulk-modulus-related response.

The corresponding current moduli can therefore be written as

$$G_c = (1 - D_s)G \quad (14)$$

$$K_c = (1 - D_v)K \quad (15)$$

which implies that the current Poisson's ratio generally becomes.

$$\nu_c = \frac{3K_c - 2G_c}{2(3K_c + G_c)} \quad (16)$$

and does not remain constant unless $D_s = D_v$. This point highlights one of the main differences between the present formulation and the conventional scalar $(1-D)$ -type isotropic damage model. In the latter case, the volumetric and deviatoric stiffness contributions degrade proportionally and the associated Poisson's ratio remains constant, whereas in the present model the two constitutive channels may evolve differently under different stress states.

The present formulation recovers the classical scalar damage model as a particular case: when $D_s = D_v = D$, the two channels collapse into a single degradation process and the model reduces to the conventional $(1-D)$ -type isotropic law.

2.2.2. Effective-stress-based microscopic plasticity

The microscopic plasticity retains the effective-stress-based structure of Araújo et al. [31], extended here to the volumetric/deviatoric damage form. Yielding occurs only at the microscopic scale.

Microscopic plasticity is expressed in an effective-stress space, and a von Mises criterion is adopted as

$$\phi^{\mu} = \bar{q}^{\mu} - \sigma_f^{\infty} \quad (17)$$

where σ_f^{∞} denotes the fatigue limit of the matrix in which the micro-inclusion is embedded, and $\bar{q}^{\mu}$ is the equivalent stress defined from the effective deviatoric stress. In this setting, plasticity is expected to occur first at the microscopic scale, while the macroscopic scale remains elastic.

The effective deviatoric stress and the corresponding equivalent measure are defined as

$$\bar{q}^\mu = \sqrt{\frac{3}{2} \mathbf{r}^\mu : \mathbf{r}^\mu}, \quad \mathbf{r}^\mu = \frac{\mathbf{S}^\mu}{1 - D_s} - \boldsymbol{\alpha}^\mu \quad (18)$$

where $\mathbf{S}^\mu$ is the microscopic deviatoric stress tensor defined in Section 2.2.1 and $\boldsymbol{\alpha}^\mu$ is the microscopic backstress tensor. The quantity $\bar{q}^\mu$ represents the von Mises norm of the effective deviatoric stress. Substituting Eq. (11) into Eq. (18) yields.

$$\mathbf{r}^\mu = 2G\text{dev}(\boldsymbol{\epsilon}^{\mu e}) - \boldsymbol{\alpha}^\mu \quad (19)$$

Since the von Mises yield function depends only on deviatoric stress invariants, D_v does not enter the yield condition directly but influences the response through the volumetric elastic stiffness, the driving force Y_v , and the failure criterion.

With an associated flow rule, the microscopic plastic-strain rate is written as

$$\dot{\boldsymbol{\epsilon}}^{\mu p} = \frac{\dot{\gamma}^\mu}{1 - D_s} \sqrt{\frac{3}{2}} \frac{\mathbf{r}^\mu}{\|\mathbf{r}^\mu\|} \quad (20)$$

where $\dot{\gamma}^\mu$ is the microscopic plastic multiplier. This relation preserves the effective-stress-based flow structure of the original two-scale formulation while adapting the damage dependence to the present deviatoric damage channel.

Kinematic hardening follows the Armstrong–Frederick form,

$$\dot{\boldsymbol{\alpha}}^\mu = \frac{2}{3} H_k \dot{\boldsymbol{\epsilon}}^{\mu p} - b \frac{\dot{\gamma}^\mu}{1 - D_s} \boldsymbol{\alpha}^\mu \quad (21)$$

where H_k is the kinematic hardening modulus and b is the saturation parameter. The loading/unloading conditions and the consistency condition satisfy the Kuhn–Tucker constraints,

$$\phi^\mu \leq 0, \quad \dot{\gamma}^\mu \geq 0, \quad \phi^\mu \dot{\gamma}^\mu = 0 \quad (22)$$

2.2.3. Thermodynamic driving forces, damage evolution, and fatigue failure criterion

To complete the constitutive description, the evolution of the two damage variables is formulated within a thermodynamically consistent framework. Since the microscopic elastic response has been decomposed into deviatoric and volumetric parts in Section 2.2.1, the Helmholtz free-energy density is written accordingly as

$$u(\boldsymbol{\epsilon}^{\mu e}, D_s, D_v) = G(1 - D_s)\text{dev}(\boldsymbol{\epsilon}^{\mu e}) : \text{dev}(\boldsymbol{\epsilon}^{\mu e}) + \frac{1}{2} K(1 - D_v)[\text{tr}(\boldsymbol{\epsilon}^{\mu e})]^2 \quad (23)$$

At fixed strain, partial differentiation with respect to the damage variables yields the energy release rates associated with the deviatoric and volumetric channels,

$$Y_s = - \left. \frac{\partial u}{\partial D_s} \right|_{\boldsymbol{\epsilon}^{\mu e}} = G\text{dev}(\boldsymbol{\epsilon}^{\mu e}) : \text{dev}(\boldsymbol{\epsilon}^{\mu e}) \quad (24)$$

$$Y_v = - \left. \frac{\partial u}{\partial D_v} \right|_{\boldsymbol{\epsilon}^{\mu e}} = \frac{1}{2} K[\text{tr}(\boldsymbol{\epsilon}^{\mu e})]^2 \quad (25)$$

These two driving forces may be viewed as the deviatoric and volumetric counterparts of the single-damage energy release rate used in the reference two-scale formulation. Denoting by $q^\mu = \sqrt{\frac{3}{2} \mathbf{S}^\mu : \mathbf{S}^\mu}$ the von Mises equivalent stress constructed from the microscopic Cauchy deviatoric stress $\mathbf{S}^\mu$, and using Eqs. (11) and (12), the energy release rates can be recast in terms of stress invariants as

$$Y_s = \frac{(q^\mu)^2}{6G(1 - D_s)^2}, \quad Y_v = \frac{(p^\mu)^2}{2K(1 - D_v)^2} \quad (26)$$

Since $Y_s \geq 0$ and $Y_v \geq 0$, and the two damage variables accumulate monotonically, the dissipation associated with damage evolution satisfies the Clausius–Duhem inequality,

$$\dot{D}_d = Y_s \dot{D}_s + Y_v \dot{D}_v \geq 0 \quad (27)$$

The present evolution laws preserve the Lemaitre-type structure (plastic-multiplier driving, energy-release-rate partitioning, damage-strength denominator, $s = 1$) but extend it to two independent channels. The evolution laws are written as

$$\dot{D}_s = \frac{\dot{\gamma}^\mu}{1 - D_s} \frac{Y_s}{S_s(\eta^\mu, \xi^\mu)} \quad (28)$$

$$\dot{D}_v = \frac{\dot{\gamma}^\mu}{1 - D_v} \frac{Y_v}{S_v(\eta^\mu, \xi^\mu)} \quad (29)$$

To describe differences in damage accumulation efficiency under tension–compression, torsion, and other proportional loading paths within a unified form, stress-state-dependent damage strength functions $S_s(\eta^\mu, \xi^\mu)$ and $S_v(\eta^\mu, \xi^\mu)$ are introduced. Their functional form follows the improved denominator-of-damage formulation of Malcher and Mamiya [8], in which the damage strength depends on the stress triaxiality and the normalized third invariant of the deviatoric stress tensor. In the present model, the two functions share the same structure but retain independent calibration parameters, so that the two damage channels may accumulate at different rates. A proportional constraint $S_{v,0.0}/S_{v,0.33} = S_{s,0.0}/S_{s,0.33}$ is imposed to ensure that both functions share the same stress-state variation trend while reducing the number of independent calibration parameters to three.

The two functions are written as

$$S_s = \frac{S_{s,0.33}}{3|\eta^\mu| + \frac{S_{v,0.33}}{S_{v,0.0}}(1 - (\xi^\mu)^2)} \quad (30)$$

$$S_v = \frac{S_{v,0.33}}{3|\eta^\mu| + \frac{S_{v,0.33}}{S_{v,0.0}}(1 - (\xi^\mu)^2)} \quad (31)$$

The stress-state parameters are computed from the current microscopic stress invariants as

$$\eta^\mu = \frac{p^\mu}{q^\mu}, \quad \xi^\mu = \frac{27\det(\mathbf{S}^\mu)}{2(q^\mu)^3} \quad (32)$$

where $S_{\alpha,0.33}$ and $S_{\alpha,0.0}$ with $\alpha \in \{s, v\}$ govern the damage strength levels associated with the reference tension–compression and pure-shear stress states, respectively. For a given proportional loading path, the values of η^μ and ξ^μ uniquely determine the corresponding damage strength. The two damage components thus accumulate at independently adjustable rates governed by their respective driving forces and strength functions.

To connect the evolution of the two damage components with a fatigue failure criterion, a normalized effective damage index is introduced. In the present dual-damage framework, failure is defined in a linear interaction form as

$$D_{ef} = \frac{D_s}{D_{s,c}} + \frac{D_v}{D_{v,c}} \geq 1 \quad (33)$$

where $D_{s,c}$ and $D_{v,c}$ denote the critical values associated with the deviatoric and volumetric damage channels, respectively. In the present work, an equal-threshold assumption is adopted in order to avoid introducing additional failure parameters,

$$D_{s,c} = D_{v,c} = D_c \quad (34)$$

The critical damage level is approximated from monotonic tensile properties as

$$D_c = 1 - \frac{\sigma_F}{\sigma_u} \quad (35)$$

Under this assumption, the failure condition in Eq. (33) reduces to $D_s + D_v \geq D_c$. This criterion represents an extension of the critical-damage concept employed in the reference two-scale formulation to the present dual-damage setting. The equal-threshold assumption is a first-implementation simplification; distinct critical values may be

considered in future work.

Once the critical condition is satisfied at the end of a loading cycle, the corresponding number of cycles is taken as the predicted fatigue life N_f . The roles of the two damage channels under representative loading paths are discussed in Section 3.3. The complete logical structure of the coupled volumetric/deviatoric damage elasto-plastic constitutive model is illustrated in Fig. 2.

2.3. Numerical implementation

The constitutive model formulated in Sections 2.2 is path-dependent, and an incremental numerical procedure is therefore required to advance the internal variables over each loading cycle. In the present work, the stress-update procedure is based on the operator-split concept, in which the problem is decomposed into an elastic predictor stage and a plastic corrector stage. Each loading cycle is discretized into $M = 20$ equally spaced pseudo-time increments. Since the macroscopic scale remains elastic throughout (Section 2.1), the macroscopic strain increment at each step is determined directly from the prescribed stress through the elastic constitutive relation. The numerical procedure is implemented in MATLAB.

For a given macroscopic strain ϵ and the set of state variables at pseudo-time t_n , the goal is to obtain the updated state at t_{n+1} . The trial microscopic elastic strain is first assembled under the assumption that the strain increment is entirely elastic:

$$\epsilon_{\text{trial}}^{\mu e} = \epsilon + (a - 1)\epsilon_n^{\mu p} \quad (36)$$

The trial effective deviatoric stress and the corresponding equivalent stress are then computed:

$$\mathbf{r}_{\text{trial}} = 2G\text{dev}(\epsilon_{\text{trial}}^{\mu e}) - \alpha_n, \quad \bar{q}_{\text{trial}} = \sqrt{\frac{3}{2}\mathbf{r}_{\text{trial}} : \mathbf{r}_{\text{trial}}} \quad (37)$$

If the trial state satisfies the microscopic yield condition, i.e. $\bar{q}_{\text{trial}} \leq \sigma_f^\infty$, the elastic assumption holds and the trial state is accepted as the solution at t_{n+1} . Otherwise, the yield condition is violated and a plastic corrector must be applied. In the plastic correction stage, a radial-return approximation is adopted, in which the flow direction is frozen at the trial state:

$$\mathbf{n} = \frac{3}{2} \frac{\mathbf{r}_{\text{trial}}}{\bar{q}_{\text{trial}}} \quad (38)$$

The plastic-strain increment and the accumulated plastic strain are updated as:

$$\Delta \epsilon^{\mu p} = \frac{\Delta \gamma}{1 - D_{s,n+1}} \mathbf{n}, \quad \epsilon_{n+1}^{\mu p} = \epsilon_n^{\mu p} + \Delta \epsilon^{\mu p} \quad (39)$$

The backstress is updated according to the implicit discretization of the Armstrong–Frederick law:

$$\alpha_{n+1}^\mu = \frac{\alpha_n^\mu + \frac{2}{3}H_k \Delta \epsilon^{\mu p}}{1 + b \frac{\Delta \gamma}{1 - D_{s,n+1}}} \quad (40)$$

After the plastic correction, the microscopic elastic strain is reconstructed from the updated variables, and the microscopic Cauchy stress is recomputed. The stress-state parameters (η^μ , ξ^μ), the energy release rates (Y_s , Y_v), and the damage strength functions $S_s(\eta^\mu, \xi^\mu)$ and $S_v(\eta^\mu, \xi^\mu)$ are then evaluated at the updated state. The two damage variables are advanced using the implicit discrete forms of the evolution laws established in Section 2.2.3:

$$D_{s,n+1} = D_{s,n} + \frac{\Delta \gamma}{1 - D_{s,n+1}} \frac{Y_{s,n+1}}{S_{s,n+1}}, \quad D_{v,n+1} = D_{v,n} + \frac{\Delta \gamma}{1 - D_{v,n+1}} \frac{Y_{v,n+1}}{S_{v,n+1}} \quad (41)$$

Since Eqs. (39)–(41) are fully implicit and coupled, the plastic multiplier increment $\Delta \gamma$, the deviatoric damage increment ΔD_s , and the volumetric damage increment ΔD_v must be determined simultaneously. The unknowns are collected into a vector:

$$\mathbf{x} = \{\Delta \gamma, \Delta D_s, \Delta D_v\}^T \quad (42)$$

and a system of three nonlinear residual equations is constructed from the yield consistency condition and the two damage evolution relations:

$$\mathbf{R}(\mathbf{x}) = \begin{bmatrix} R_f \\ R_s \\ R_v \end{bmatrix} = \begin{bmatrix} \bar{q}_{n+1}(\mathbf{x}) - \sigma_f^\infty \\ \Delta D_s - \frac{\Delta \gamma}{1 - D_{s,n+1}} \frac{Y_{s,n+1}(\mathbf{x})}{S_{s,n+1}(\mathbf{x})} \\ \Delta D_v - \frac{\Delta \gamma}{1 - D_{v,n+1}} \frac{Y_{v,n+1}(\mathbf{x})}{S_{v,n+1}(\mathbf{x})} \end{bmatrix} = \mathbf{0} \quad (43)$$

This nonlinear system is solved by means of Newton–Raphson iterations:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} - [\mathbf{J}^{(k)}]^{-1} \mathbf{R}(\mathbf{x}^{(k)}), \quad \mathbf{J}^{(k)} = \frac{\partial \mathbf{R}}{\partial \mathbf{x}} \bigg|_{\mathbf{x}^{(k)}} \quad (44)$$

where $\mathbf{J}^{(k)}$ denotes the Jacobian matrix evaluated at the current iterate. The iteration is terminated when the Euclidean norm of the residual vector satisfies $\|\mathbf{R}\| < 10^{-10}$, with a maximum of 15 iterations permitted per increment.

Once the constitutive update has converged at all increments within

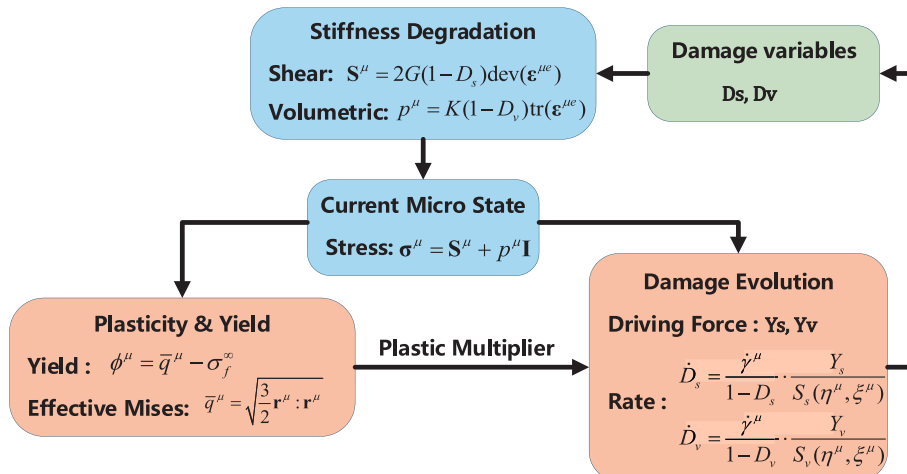


Fig. 2. Diagram of the coupled volumetric/deviatoric damage elasto-plastic constitutive model.

the current cycle, the fatigue life prediction proceeds by cycle-by-cycle integration. At the end of each loading cycle, the failure condition established in Eq. (33) is evaluated through the failure indicator:

$$\Phi = \frac{D_s}{D_{s,c}} + \frac{D_v}{D_{v,c}} - 1 \quad (45)$$

The loading process continues cycle by cycle until $\Phi \geq 0$, at which point the current cycle number is registered as N_f .

Parameter calibration involves only three parameters in the damage strength functions, $\theta = (S_{s,0.33}, S_{s,0.0}, S_{v,0.33})$, and a proportional constraint $S_{v,0.0}/S_{v,0.33} = S_{s,0.0}/S_{s,0.33}$ is imposed to reduce the degrees of freedom. The parameters are determined by minimizing the root-mean-square error between predicted and experimental lives on a logarithmic scale:

$$\text{RMSE}(\theta) = \sqrt{\frac{1}{N} \sum_{i=1}^N [\log_{10} N_{\text{pred}}^{(i)}(\theta) - \log_{10} N_{\text{exp}}^{(i)}]^2} \quad (46)$$

A genetic algorithm is employed for global search to reduce the risk of convergence to local optima, followed by local gradient-based refinement. Except for θ , all other material parameters are held fixed across all loading paths and all data points to ensure comparability of predictions and stability of parameter identification. The flowchart of the implicit return-mapping algorithm is shown in Fig. 3.

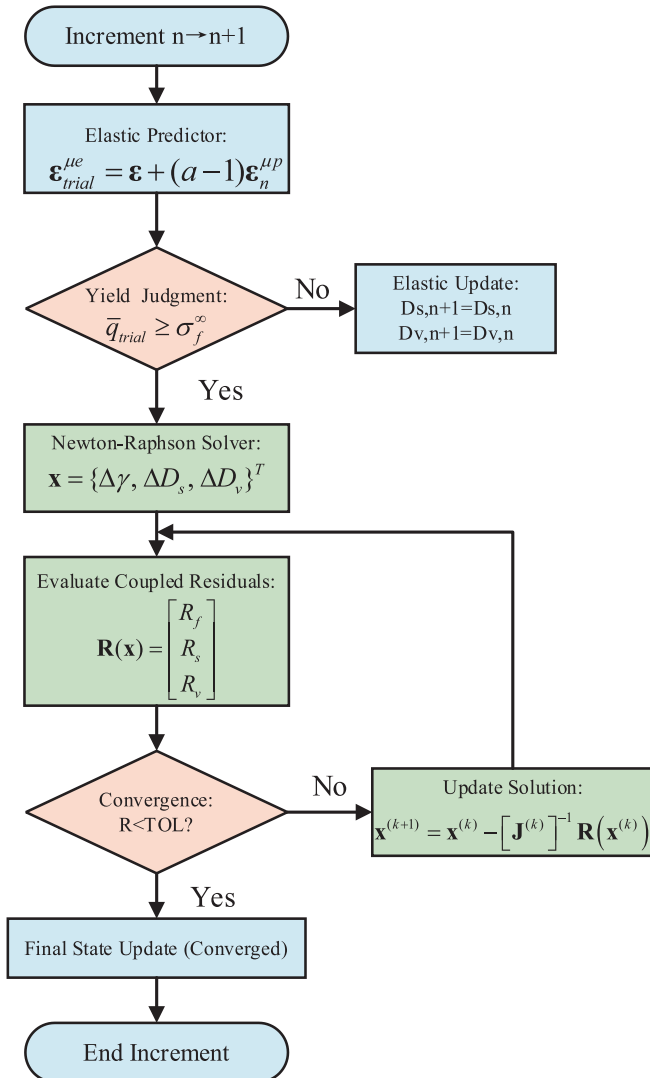


Fig. 3. Flowchart of the implicit return-mapping algorithm.

3. Model parameter identification and analysis

3.1. Parameter identification

This section calibrates and validates the model using publicly available multiaxial constant-amplitude fatigue data for 7050-T7451 aluminum alloy [31]. Owing to its high specific strength, this alloy is widely used in aerospace structures, and its high-cycle fatigue response exhibits pronounced stress-state sensitivity and scatter, making it a suitable benchmark material for evaluating multiaxial fatigue prediction frameworks. The material properties are summarized in Table 1.

The experiments cover three representative loading paths, namely axial tension–compression (Path A), fully reversed pure torsion (Path B), and proportional biaxial loading (Path C), as shown in Fig. 4. For Path C, the shear-to-normal stress-amplitude ratio is controlled by $\lambda = \sqrt{3}\tau_a/\sigma_a$, with $\lambda = \sqrt{3}/2$, $3\sqrt{3}/20$ and $2\sqrt{3}$ used in the tests. For conditions with lives on the order of 10^4 cycles, macroscopic hysteresis remains negligible and the macroscopic stress state stays within the elastic regime, confirming the applicability premise of the present two-scale elastic/elasto-plastic framework. The complete datasets are provided in Tables 2–4, comprising 41 data points with lives spanning approximately three orders of magnitude (2.86×10^4 to 9.47×10^6 cycles).

The parameters to be identified are $\theta = (S_{s,0.33}, S_{s,0.0}, S_{v,0.33})$, while $S_{v,0.0}$ is derived from $S_{v,0.0}/S_{v,0.33} = S_{s,0.0}/S_{s,0.33}$. Parameter identification minimizes the logarithmic-life RMSE (Eq. (46)). GA is used for global search and fmincon is then applied for local refinement. The search ranges are $S_{s,0.33} \in [200, 3000]$, $S_{s,0.0} \in [1000, 5000]$, $S_{v,0.33} \in [30, 1000]$, with a population size of 50 and 30 generations. Ten independent runs with different random seeds yield an objective scatter of only 1.8%, indicating adequate convergence. Except for θ , all other material parameters are held fixed to ensure comparability and identification stability.

The optimal solution is $\theta^* = (S_{s,0.33} = 2648.4 \text{ MPa}, S_{s,0.0} = 3385.6 \text{ MPa}, S_{v,0.33} = 151.5 \text{ MPa})$, with $S_{v,0.0} = 193.7 \text{ MPa}$ derived from the proportionality constraint.

These calibrated quantities should be interpreted as stress-state-dependent damage strength parameters rather than as arbitrary fitting constants. $S_{s,0.33}$ governs the resistance to deviatoric damage accumulation under the reference uniaxial tension–compression stress state ($\eta = 1/3$), while $S_{s,0.0}$ governs the deviatoric damage resistance under pure shear ($\eta = 0$). The calibrated ratio $S_{s,0.0}/S_{s,0.33} = 1.278$ indicates that the material exhibits higher resistance to shear-driven damage under torsion than under tension, which is consistent with the lower effective contribution of hydrostatic stress under low-triaxiality conditions. $S_{v,0.33}$ governs the resistance to volumetric damage accumulation under uniaxial tension, and $S_{v,0.0}$ follows the same proportionality relation for the shear-dominated reference state.

The relative magnitudes of the calibrated parameters are physically consistent. The volumetric damage strength is substantially lower than the deviatoric damage strength, reflecting the different scales of the energy release rates Y_s and Y_v , so that a correspondingly lower damage

Table 1
Material properties of aluminum alloy 7050-T7451.

Parameter	Symbol	Value	Parameter	Symbol	Value
Young's modulus	E	73.4 GPa	Saturation constant	b	100.88
Poisson's ratio	ν	0.3	Tensile strength	σ_u	536.0 MPa
Initial yield strength	σ_{y0}	306.7 MPa	Fracture stress	σ_F	483.0 MPa
Fatigue limit (10^7 cycles)	σ^∞	100.0 MPa	Critical damage	D_c	0.1
Kinematic hardening modulus	H^k	6035.68 MPa		\	\

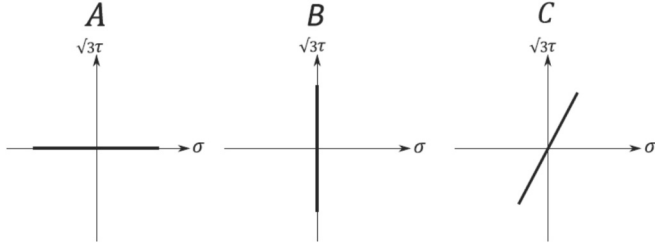


Fig. 4. Typical loading paths: (A) axial tension-compression; (B) pure torsion; (C) biaxial proportional.

Table 2

Axial tension-compression fatigue data (Path A).

σ_a (MPa)	τ_a (MPa)	N_{EXP} (cycles)	σ_a (MPa)	τ_a (MPa)	N_{EXP} (cycles)
205.0	0.0	7.80×10^4	146.0	0.0	1.45×10^5
205.0	0.0	4.58×10^4	129.0	0.0	9.77×10^5
180.0	0.0	6.57×10^4	129.0	0.0	3.00×10^5
180.0	0.0	5.26×10^4	129.0	0.0	1.47×10^5
180.0	0.0	1.19×10^5	112.0	0.0	8.23×10^6
180.0	0.0	9.49×10^4	112.0	0.0	2.92×10^5
180.0	0.0	5.45×10^4	112.0	0.0	1.27×10^6
146.0	0.0	2.13×10^5	\	\	\

Table 3

Torsion fatigue data (Path B).

σ_a (MPa)	τ_a (MPa)	N_{EXP} (cycles)	σ_a (MPa)	τ_a (MPa)	N_{EXP} (cycles)
0.0	173.2	8.90×10^4	0.0	86.6	2.42×10^6
0.0	173.2	5.36×10^4	0.0	86.6	2.07×10^6
0.0	173.2	2.86×10^4	0.0	79.7	2.50×10^6
0.0	118.4	3.68×10^5	0.0	79.7	2.35×10^6
0.0	108.6	1.54×10^6	0.0	67.7	1.74×10^6
0.0	108.6	6.45×10^5	0.0	67.7	6.23×10^6
0.0	108.6	5.00×10^5	0.0	67.7	9.47×10^6
0.0	86.6	3.24×10^6	0.0	67.7	5.11×10^6
0.0	86.6	2.76×10^6	\	\	\

strength denominator is required to produce physically meaningful volumetric damage rates. The proportional constraint $S_{v,0.0}/S_{v,0.33} = S_{s,0.0}/S_{s,0.33}$ ensures that both damage channels share the same stress-state variation trend, reducing the number of free parameters to three while maintaining a physically consistent dual-channel structure.

3.2. Accuracy assessment and ablation study

3.2.1. Calibration accuracy

Using the optimal parameter set θ^* , full cycle-by-cycle integration is performed for all 41 data points. The comparison between the predicted life N_{pred} and the experimental life N_{exp} is shown in Fig. 5. Both axes are in logarithmic scale; the solid line denotes perfect agreement, while the dashed and dash-dotted lines indicate the factor-of-2 and factor-of-4 scatter bands, respectively. The predictions for all three paths are evenly distributed about the agreement line, with no systematic over- or under-prediction. This indicates that the stress-state-dependent damage strength functions $S_s(\eta^\mu, \xi^\mu)$ and $S_v(\eta^\mu, \xi^\mu)$ can adapt across axial,

torsional, and proportional combined stress states, effectively overcoming the inherent limitation of fixed-constant S schemes, in which predictions for different paths are mutually inconsistent.

The accuracy metrics are summarized in Table 5. The global prediction accuracy reaches $F_2 = 63.4\%$ and $F_4 = 90.2\%$. The F_4 values for the three paths are 93.3% (Path A), 88.2% (Path B), and 88.9% (Path C), respectively. Only four of the 41 data points fall outside the factor-of-4 scatter band. These outliers are attributable to intrinsic material scatter near the fatigue limit (the 112 MPa axial point, where replicate specimens exhibit a 28-fold life spread), known limitations of the Armstrong-Frederick hardening formulation at high stress (the 173.2 MPa torsion points), and inherently large scatter in the near-threshold high-cycle region (the 42.1/69.5 MPa proportional point). For reference, Araújo et al. [31] reported that the SWT criterion achieved $F_4 = 92.7\%$ on the same dataset. Unlike the present model, however, SWT does not provide mechanism-resolved damage information.

3.2.2. Ablation study

This section isolates the contribution of each constitutive innovation through a controlled ablation study performed within a unified numerical framework. The two targeted features are the stress-state-dependent damage-strength functions and the separation of damage into independent deviatoric and volumetric components (D_s, D_v). Three control schemes are constructed by degrading the present model, such

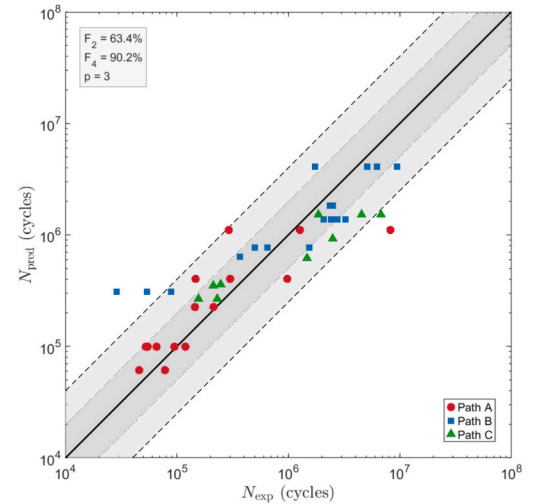


Fig. 5. Comparison between predicted and experimental lives for the calibrated dataset.

Table 5

Statistics for the calibration dataset.

Path	Number	F_2 (%)	F_4 (%)
Path A	15	73.3	93.3
Path B	17	58.8	88.2
Path C	9	55.6	88.9
Overall	41	63.4	90.2

Table 4

Biaxial proportional loading fatigue data (Path C).

σ_a (MPa)	τ_a (MPa)	λ	N_{EXP} (cycles)	σ_a (MPa)	τ_a (MPa)	λ	N_{EXP} (cycles)
121.9	61.0	$\sqrt{3}/2$	1.56×10^5	42.1	69.5	$33\sqrt{3}/20$	1.86×10^6
121.9	61.0	$\sqrt{3}/2$	2.30×10^5	42.1	69.5	$33\sqrt{3}/20$	4.54×10^6
64.6	106.6	$33\sqrt{3}/20$	2.47×10^5	42.1	69.5	$33\sqrt{3}/20$	6.78×10^6
53.5	88.2	$33\sqrt{3}/20$	1.47×10^6	57.5	115.0	$2\sqrt{3}$	2.12×10^5
47.5	78.3	$33\sqrt{3}/20$	2.50×10^6	\	\	\	\

that all schemes share the same numerical solver, increment discretization ($M = 20$ sub-increments per cycle), Newton–Raphson convergence criterion (tolerance 10^{-10}), and common elastic–plastic material constants. The differences among schemes lie in the constitutive damage formulation and its calibrated damage-related parameters, ensuring that any observed accuracy variation can be attributed primarily to the model structure rather than to numerical implementation.

The three control schemes are defined as follows. The fixed-S tension scheme adopts a single scalar damage variable ($D_s = D_v = D$) with a constant damage strength S calibrated from Path A data alone. The fixed-S torsion scheme uses the same scalar formulation, but calibrates S from Path B data alone. The variable-S scheme retains a single scalar damage variable while replacing the constant strength by a stress-state-dependent function $S(\eta, \xi)$ with two free parameters ($S_{0.33}, S_{0.0}$), calibrated over all 41 data points in line with the Malcher–Araújo development [27,31]. Together with the present model, these configurations form a progressive hierarchy from fixed-S scalar CDM, through stress-state-dependent scalar CDM, to the present dual-damage formulation.

The quantitative accuracy metrics for all four schemes are compiled in Table 6, including the calibrated parameter values, global and per-path F_2 and F_4 percentages, and the logarithmic-life RMSE.

The predicted-versus-experimental life scatter plots for the three control schemes are presented in Figs. 6–8, each using the same format as Fig. 5 with path-colored markers. The present model result has already been shown in Fig. 5.

For the tension-calibrated fixed-S scalar CDM ($S = 508.4$ MPa), the damage strength is determined exclusively from the 15 axial tension–compression specimens, and the calibrated value is then applied to all 41 data points across the three loading paths. The predicted-versus-experimental life comparison is shown in Fig. 6.

When the damage strength is fixed to a constant calibrated from the tensile path, the axial predictions remain well captured (Path A $F_4 = 93.3\%$), as the calibration data themselves are adequately fitted. However, the torsion predictions deteriorate dramatically. Path B F_4 drops to 29.4%, with 12 of the 17 torsional data points falling outside the factor-of-4 band. All torsional points are systematically underpredicted, i.e., the model assigns substantially shorter lives than experimentally observed. Path C likewise suffers ($F_4 = 44.4\%$). The global accuracy is $F_4 = 56.1\%$ with $\text{RMSE} = 0.7018$. A small S value calibrated from tension data yields excessive damage accumulation under pure shear, leading to premature predicted failure for torsional specimens.

For the torsion-calibrated fixed-S scalar CDM ($S = 2977.2$ MPa), the same single-scalar formulation is adopted, but the damage strength is calibrated from the 17 pure torsion specimens instead. The resulting life predictions are compared with the experimental data in Fig. 7.

The pattern is precisely reciprocal to the fixed-S tension scheme. Path B is now well described ($F_4 = 88.2\%$), but Path A collapses to $F_4 = 20.0\%$, with 12 of the 15 axial data points falling outside the factor-of-4 band, all systematically overpredicted. The large S value calibrated from torsion data suppresses damage accumulation under uniaxial tension, causing the model to assign substantially longer lives than experimentally observed. Path C is reasonably captured ($F_4 = 88.9\%$) due to the dominant shear component in the proportional loading. The global accuracy is $F_4 = 63.4\%$ with $\text{RMSE} = 0.6080$. The nearly six-fold difference between the two calibrated S values (508.4 MPa in the fixed-S tension scheme versus 2977.2 MPa in the fixed-S torsion scheme) quantitatively

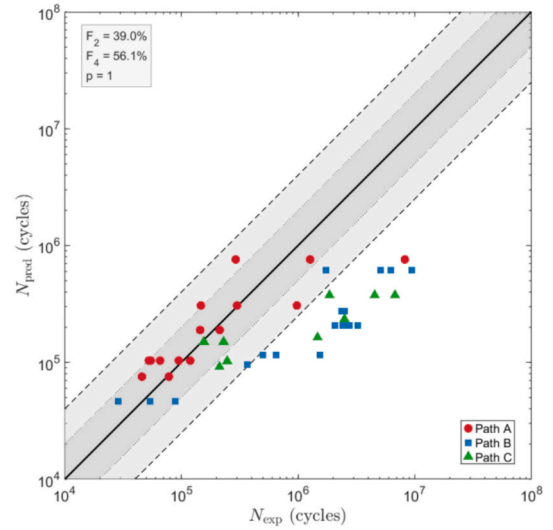


Fig. 6. Predicted versus experimental lives for the fixed-S tension scheme (scalar D , $S = S_{\text{tension}}$).

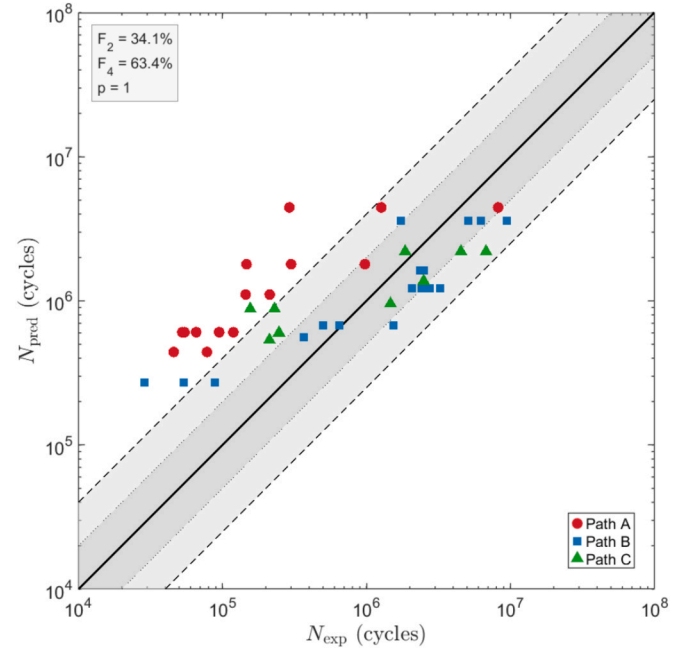


Fig. 7. Predicted versus experimental lives for the fixed-S torsion scheme (scalar D , $S = S_{\text{torsion}}$).

confirms the strong path dependence of the fixed-S formulation, consistent with findings reported by Malcher and Mamiya [27].

For the stress-state-dependent scalar CDM with $S(\eta, \xi)$, two parameters ($S_{0.33} = 680.0$ MPa, $S_{0.0} = 2750.0$ MPa) are calibrated over all 41 data points, allowing the damage strength to vary continuously with the stress triaxiality and the normalized third invariant. The predicted-

Table 6

Ablation study: accuracy comparison of four CDM schemes.

Method	Params	Calibrated values	Path A F_4	Path B F_4	Path C F_4	Global F_2	Global F_4	RMSE
fixed S, tension	1	$S = 508.4$	93.3%	29.4%	44.4%	39.0%	56.1%	0.7018
fixed S, torsion	1	$S = 2977.2$	20.0%	88.2%	88.9%	34.1%	63.4%	0.6080
variable S	2	$S_{0.33} = 680.0$ $S_{0.0} = 2750.0$	93.3%	88.2%	66.7%	53.7%	85.4%	0.4061
Present model	3	θ^*	93.3%	88.2%	88.9%	63.4%	90.2%	0.3699

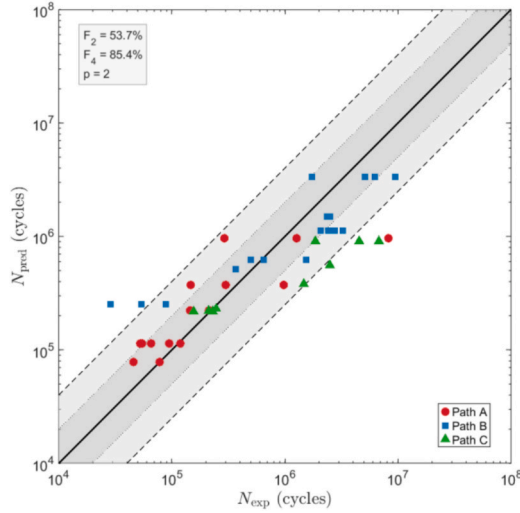


Fig. 8. Predicted versus experimental lives for the variable-S scheme (scalar D , $S(\eta, \xi)$).

versus-experimental life comparison for this configuration is presented in Fig. 8.

By introducing stress-state dependence through the $S(\eta, \xi)$ function, the reciprocal failure pattern observed in the two fixed-S schemes is eliminated. Path A and Path B are simultaneously well captured ($F_4 = 93.3\%$ and 88.2% , respectively), and the global F_4 rises to 85.4% with $\text{RMSE} = 0.4061$. However, Path C remains at $F_4 = 66.7\%$, with three of the nine proportional-loading data points falling outside the factor-of-4 band. This indicates that although the $S(\eta, \xi)$ formulation resolves the tension-torsion discrepancy, it does not fully accommodate damage evolution under intermediate mixed stress states where both deviatoric and volumetric mechanisms are simultaneously active. Under proportional loading, the single scalar driving force $Y = Y_s + Y_v$ merges the deviatoric and volumetric contributions into a single rate equation, which inherently cannot capture their differential evolution.

The present model (Fig. 5 and Table 6) further improves Path C from $F_4 = 66.7\%$ (stress-state-dependent scalar CDM) to 88.9% , a gain of 22.2 percentage points, while maintaining the same accuracy on Path A (93.3%) and Path B (88.2%). The global F_4 increases from 85.4% to 90.2% , and the RMSE decreases from 0.4061 to 0.3699 . This improvement is attributable to the independent evolution of D_s and D_v . Under proportional loading, both the deviatoric and volumetric driving forces are simultaneously non-zero, and the two damage components evolve at different rates governed by their respective strength functions $S_s(\eta, \xi)$ and $S_v(\eta, \xi)$. As shown in Section 3.3, the damage partitioning under Path C yields $r_v = 37.4\%$, confirming that both mechanisms contribute substantially and that a single-scalar representation is structurally insufficient.

In summary, the ablation study establishes a clear progressive improvement chain. The two fixed-S scalar CDM configurations achieve F_4 of 56.1% and 63.4% , respectively. The stress-state-dependent scalar CDM raises the global F_4 to 85.4% , and the present model further improves it to 90.2% . Each step corresponds to the activation of a specific constitutive innovation. The stress-state-dependent $S(\eta, \xi)$ resolves the path-dependence problem inherent in fixed-S scalar CDM, while the deviatoric and volumetric damage separation provides the additional capability to capture mixed-mode damage evolution under intermediate stress states.

3.3. Damage mechanism characterization

Beyond fatigue-life prediction, the deviatoric/volumetric decomposition framework provides mechanism-resolved information. This

section illustrates that capability through three complementary analyses: terminal damage partitioning at failure, damage-evolution trajectories over the fatigue life, and continuous mapping of the mechanism transition across the stress-state spectrum.

3.3.1. Terminal damage partitioning

To quantitatively characterize damage mechanisms under different loading paths, two scalar indicators are introduced, namely a damage-partitioning index (A-index) and the volumetric damage fraction r_v , defined as

$$\text{A-index} = (D_s - D_v) / (D_s + D_v), \quad r_v = D_v / (D_s + D_v).$$

An A-index approaching $+1$ indicates fully deviatoric (shear-driven) dominance, whereas an A-index approaching -1 indicates fully volumetric dominance associated with void nucleation and growth. For the constant-amplitude proportional loading paths considered here, the stress-state invariants (η, ξ) remain constant over the cycle. The ratio of driving forces for the two damage components is therefore uniquely determined by the path stress state, so that a single representative value at failure characterizes the damage partitioning of the corresponding specimen set. The damage partitioning at failure for the three paths is summarized in Table 7.

Fig. 9 visualizes these results in the D_s - D_v plane. The diagonal line denotes the failure constraint $D_s + D_v = D_c$. Path A ($n=15$) lies near the D_v axis, Path B ($n=17$) falls on the D_s axis, and Path C ($n=9$) is positioned between the two. The representative points for all three paths lie on the failure line, demonstrating the internal consistency of the deviatoric/volumetric decomposition framework with the adopted failure criterion.

The locations of the three representative points reveal distinctly different microscopic failure mechanisms. For Path B (pure torsion), the hydrostatic pressure vanishes, eliminating the volumetric driving force. Across all 17 torsional data points, the simulations yield $D_v = 0$ and purely deviatoric failure (A-index $= +1.000$), numerically confirming the correctness of the mechanism separation. For Path A (uniaxial axial loading), the dominant normal-stress component produces significant hydrostatic tension, giving $r_v = 80.2\%$ and A-index $= -0.600$, which indicates that void nucleation and growth constitute the primary dissipation mechanism, with a residual $D_s = 0.020$ attributed to shear-type microcrack extension induced by cyclic plastic strain. The representative point for Path C (in-phase proportional loading) lies near the mid-segment of the constraint line (A-index $= +0.253$, $r_v = 37.4\%$), quantitatively reflecting mixed-mode competition under intermediate stress states, consistent with the continuous stress-state transition from Path A to Path B.

3.3.2. Dynamic damage evolution trajectories

While Section 3.3.1 characterizes the damage partition only at failure, the trajectory plots reveal how the two channels compete throughout the fatigue life. Figs. 10–12 present the normalized damage-evolution curves (D/D_c versus number of cycles N) for one representative loading condition from each path, namely $\sigma_a = 180$ MPa for Path A, $\tau_a = 108.6$ MPa for Path B, and $\sigma_a = 53.5$ MPa / $\tau_a = 88.2$ MPa for Path C. In each figure, the solid black curve denotes D_{tot}/D_c , the dashed red curve denotes D_v/D_c , and the dotted blue curve denotes D_s/D_c . Where both damage components are non-negligible, the shaded region between D_{tot} and D_v indicates the deviatoric-damage contribution.

Fig. 10 shows the damage evolution under Path A (axial loading). The volumetric damage D_v grows substantially faster than the deviatoric damage D_s throughout the fatigue life. The two curves separate early and

Table 7
Damage partitioning at failure for the three loading paths.

Path	D_s	D_v	A-index	$r_v(\%)$	Dominant Mechanism
Path A	0.020	0.080	-0.600	80.2	Volumetric (void growth)
Path B	0.100	0.000	$+1.000$	0.0	Deviatoric (shear banding)
Path C	0.063	0.037	$+0.253$	37.4	Mixed mode

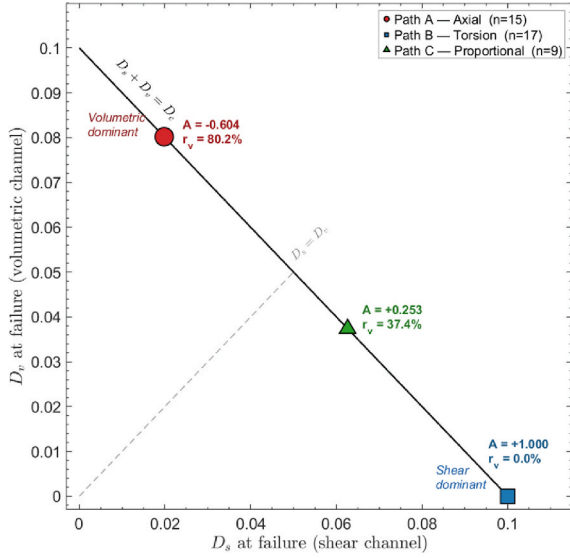


Fig. 9. Locations of the three loading paths in the damage space (D_s - D_v).

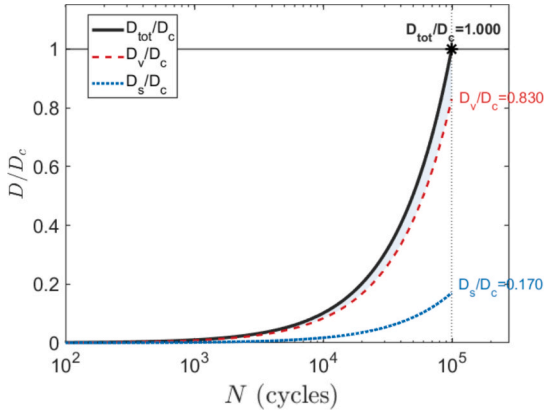


Fig. 10. Damage evolution under Path A: axial loading ($\sigma_a=180$ MPa).

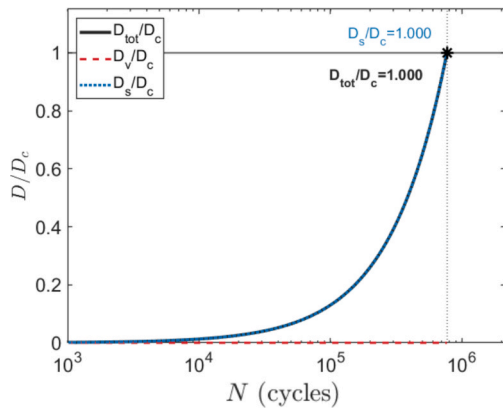


Fig. 11. Damage evolution under Path B: pure torsion ($\tau_a=108.6$ MPa).

diverge progressively, with D_v consistently accounting for the major portion of the total damage. At failure, $D_v/D_c = 0.830$ while $D_s/D_c = 0.170$. This reflects the sustained hydrostatic tension under the uniaxial stress state, which continuously drives void nucleation and growth as the dominant dissipation mechanism. The deviatoric component, although subordinate, still provides a non-negligible contribution to the final failure process.

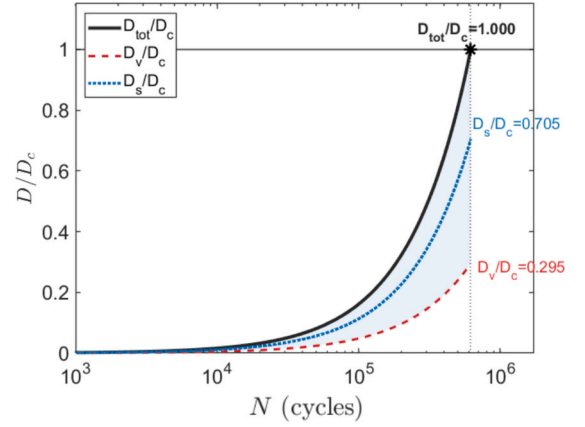


Fig. 12. Damage evolution under Path C: proportional biaxial loading ($\sigma_a=53.5$ MPa, $\tau_a=88.2$ MPa).

Fig. 11 presents the damage evolution under Path B (pure torsion). The volumetric damage D_v remains identically zero throughout the entire fatigue life, while D_s increases monotonically until reaching the failure threshold D_c . The three curves collapse into a single trajectory since $D_{tot}=D_s$. This is the direct numerical consequence of zero hydrostatic pressure under pure shear: the volumetric driving force vanishes because $p=0$, and therefore $dD_v=0$ at every increment. The model thus automatically produces a purely shear-driven failure mode without any ad hoc switching criterion.

Fig. 12 shows the damage evolution under Path C (proportional biaxial loading). Both D_s and D_v grow simultaneously but at different rates, with D_s evolving faster than D_v . At failure, $D_s/D_c=0.705$ and $D_v/D_c=0.295$, yielding $r_v=29.5\%$ for this representative case. Compared with Path A (volumetric-dominated) and Path B (purely deviatoric), Path C exhibits genuine mixed-mode behavior: both mechanisms are simultaneously active and contribute to the total damage in proportions dictated by the intermediate stress-state invariants. This mixed-mode evolution cannot be captured by a single-scalar damage model, which records only the total accumulated damage without resolving the relative contributions of shear-type and volumetric mechanisms. The ability to resolve this competition is directly related to the improved Path C accuracy achieved by the present model relative to the stress-state-dependent scalar CDM in Section 3.2.

Taken together, Figs. 10–12 show that the present framework does not merely decompose the terminal failure state at N_f , but continuously tracks the competition between shear-dominated and void-growth-dominated degradation mechanisms over the entire fatigue life.

3.3.3. Continuous stress-state dependence

The terminal partitioning discussed in Section 3.3.1 characterizes only the three discrete stress states corresponding to the available experimental paths. To verify that the transition between deviatoric- and volumetric-dominated damage is continuous and physically consistent across the broader stress-state spectrum, a parametric sweep is performed over the stress triaxiality η from 0 (pure shear) to 1/3 (uniaxial tension).

At each η value, a proportional loading condition is constructed by combining axial and shear stress amplitudes such that the von Mises equivalent stress amplitude is held constant at $\sigma_{eq}=180$ MPa while the stress triaxiality varies from $\eta = 0$ (pure shear) to $\eta = 1/3$ (uniaxial tension). The model is integrated to failure at each of the 21 equally spaced η values to obtain the terminal damage partitioning. The resulting r_v - η relation is shown in Fig. 13.

The positions of Path A, Path B, and the three proportional sub-paths of Path C are marked on the curve. It should be noted that the Path A marker in Fig. 13 denotes the path-averaged terminal partition $r_v=80.2\%$ from Table 7, which represents the average over all axial

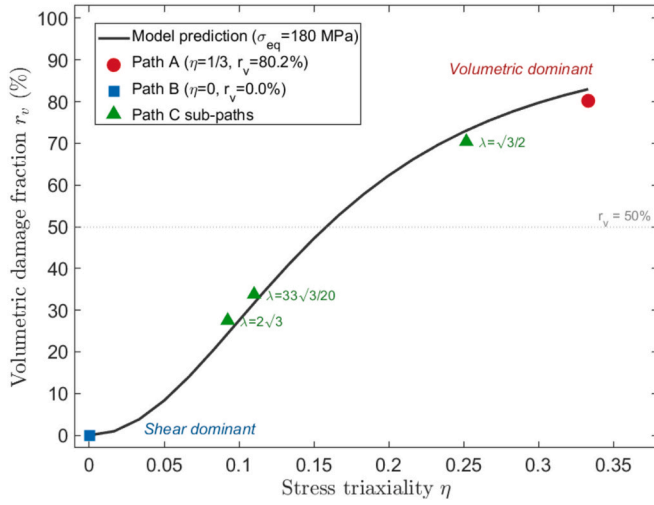


Fig. 13. Volumetric damage fraction r_v as a function of stress triaxiality η .

stress amplitudes. This value is slightly lower than the sweep endpoint $r_v=83.0\%$, which corresponds to the representative axial case $\sigma_a=180$ MPa used in Fig. 10. The difference reflects a mild stress-amplitude dependence of the damage partition within the same loading family.

The r_v - η curve exhibits a smooth and monotonically increasing trend from $r_v=0\%$ at $\eta=0$ to $r_v=83.0\%$ at $\eta=1/3$. At $\eta=0$, the volumetric damage fraction is exactly zero, consistent with the Path B result. As η increases, the hydrostatic contribution grows relative to the deviatoric component, and the volumetric damage fraction rises accordingly. The transition is continuous, with no abrupt jumps or non-monotonic behavior.

The crossover point where $r_v=50\%$ occurs at approximately $\eta \approx 0.167$, corresponding to a proportional loading condition with approximately balanced axial and shear contributions. Below this value, failure is predominantly shear-driven ($D_s > D_v$); above it, volumetric damage becomes dominant ($D_v > D_s$). This crossover therefore establishes a physically meaningful boundary between shear-dominated and void-growth-dominated fatigue regimes.

The three Path C sub-paths are located at $\eta = 0.252$ ($\lambda=\sqrt{3}/2$, $r_v=70.4\%$), $\eta = 0.110$ ($33\sqrt{3}/20$, $r_v=33.8\%$), and $\eta = 0.092$ ($\lambda=2\sqrt{3}$, $r_v=27.5\%$). All three points lie on the continuous sweep curve, confirming that the partitioning predicted for the calibration data is fully consistent with the systematic η -dependence. The substantial variation in r_v across the three proportional sub-paths further demonstrates the strong sensitivity of the damage mechanism to the stress state, even within the same nominal proportional-loading category.

The continuous r_v - η mapping therefore generalizes the discrete path-wise observations to the full stress-state spectrum and complements the fatigue-life predictions with quantitative mechanism information.

4. External validation and engineering Application

This section demonstrates the engineering-level transferability of the proposed framework beyond the constitutive-level validation in Section 3. It contains two parts: external validation on an independent handbook dataset (Section 4.1) and engineering fatigue assessment with benchmarking (Section 4.2).

4.1. External validation

To evaluate the generalization capability of the calibrated parameter set θ^* beyond the calibration domain, an external validation dataset fully independent of the calibration data in Section 3 is introduced. The validation data are taken from the constant-amplitude S-N baseline

curves in MIL-HDBK-5H [34] for 7050-T7451 aluminum alloy plates in the L (longitudinal) and LT (long-transverse) orientations under a stress ratio of $R=-1$. The dataset includes 12 stress-amplitude levels for L and 11 for LT, with lives spanning approximately 10^3 – 10^6 cycles. The calibrated parameters θ^* are used directly for validation, with no parameter adjustment or refitting.

For each stress-amplitude level, full cycle-by-cycle integration is performed by driving the volumetric/deviatoric damage-decomposed CDM model under fully reversed loading ($R=-1$). The predictions for the L and LT orientations are shown in Fig. 14 and Fig. 15, respectively.

For both orientations, F_4 reaches 100%, meaning that all predictions fall within the factor-of-4 scatter band. For the L orientation, $R^2 = 0.940$ and $F_2 = 75.0\%$. The three points outside the factor-of-2 band are concentrated at the high-stress, short-life end ($N < 10^4$ cycles), where intrinsic material scatter is larger and the observed prediction deviations remain within physical expectations. For the LT orientation, $R^2 = 0.956$ and $F_2 = 100\%$, i.e., all 11 predictions fall within the factor-of-2 scatter band. This reflects the material feature that the LT S-N curve has a gentler slope and a more regular stress-life relation.

These results demonstrate that the stress-state-dependent damage strength parameters calibrated from three multiaxial proportional loading paths (axial, torsional, and proportional biaxial) remain predictive when applied to independent handbook data under purely uniaxial loading. This extrapolative validation across loading paths and data sources supports the transferability and robustness of the calibrated framework beyond the original multiaxial calibration dataset.

4.2. Engineering Application

To further evaluate the applicability of the volumetric/deviatoric damage-decomposed CDM framework under practical multiaxial loading, a key load-bearing component in an aircraft cargo-door lifting mechanism is selected as an engineering case [21]. During service, the component is subjected to repeated coupled loading induced by the door opening-closing motion. A rigid-flexible coupled transient analysis is first performed to determine the stress evolution in the component, and the fatigue-critical location is identified at the filleted transition zone near the pinhole, where stress concentration and a steep stress gradient persist during the motion cycle, as shown in Fig. 16. The local multiaxial stress history at this hot spot is then extracted in a local coordinate system and directly used as the loading input for the proposed fatigue-damage simulation. At the critical section, the stress state contains all six independent stress components, i.e. σ_x , σ_y , σ_z , τ_{xy} , τ_{yz} , τ_{xz} . The representative hot-spot stress history and phase relations among the stress components are presented in Fig. 17.

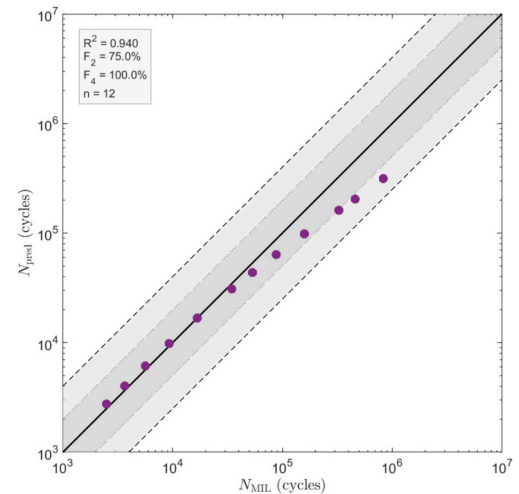


Fig. 14. MIL-HDBK-5H: L direction.

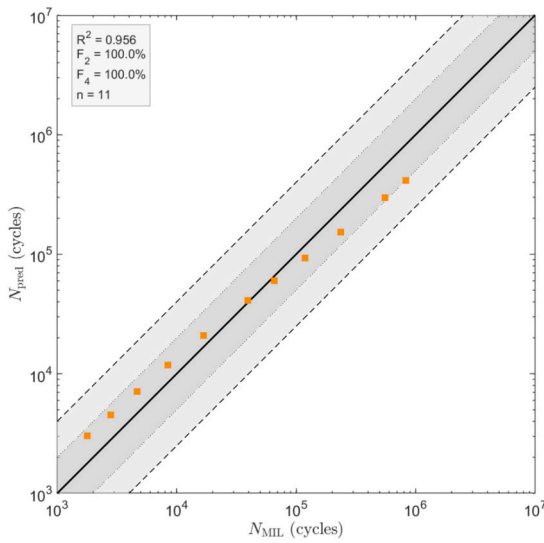


Fig. 15. MIL-HDBK-5H: LT direction.



Fig. 16. Hot-spot identification from rigid-flexible simulation.

To verify the reliability of the rigid-flexible coupled model and the extracted load input, a system-level durability test is conducted on the

cargo-door lifting mechanism. Considering the left-right symmetry of the structure, one crank on a single side is selected as the test component. Uniaxial strain gauges are arranged on the four side surfaces of the crank, as shown in Fig. 18. The measured strain signals are processed to suppress high-frequency noise and are then compared with the simulated strain responses at the corresponding locations. The comparison results in Fig. 19 show good agreement in both temporal trend and response magnitude over the full motion cycle, indicating that the rigid-flexible coupled simulation can capture the primary deformation characteristics of the component with sufficient accuracy. This validation supports the use of the extracted hot-spot multi-axial stress history as the input for the subsequent fatigue-life and damage-evolution analysis.

Based on the validated stress time histories, the six-component load history at the critical section is used as the input to the volumetric/deviatoric damage-decomposed CDM model. Full cycle-by-cycle integration is performed using the calibrated parameters θ^* directly, without introducing any path-correction factors or additional empirical parameters. The loading cycle is repeatedly applied until the total damage reaches the critical threshold, yielding a predicted fatigue life of $N_f = 1.430 \times 10^5$ cycles.

At failure, the deviatoric and volumetric damage components are $D_s = 0.0102$ and $D_v = 0.0898$, respectively. The corresponding indicators are $A\text{-index} = -0.796$ and $r_v = 89.8\%$. This partitioning shows pronounced volumetric-damage dominance, indicating that void nucleation and growth are the primary microscopic damage mechanism under the six-component engineering loading.

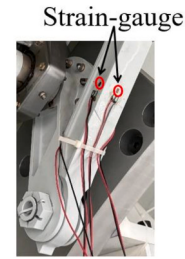


Fig. 18. Strain-gauge layout.

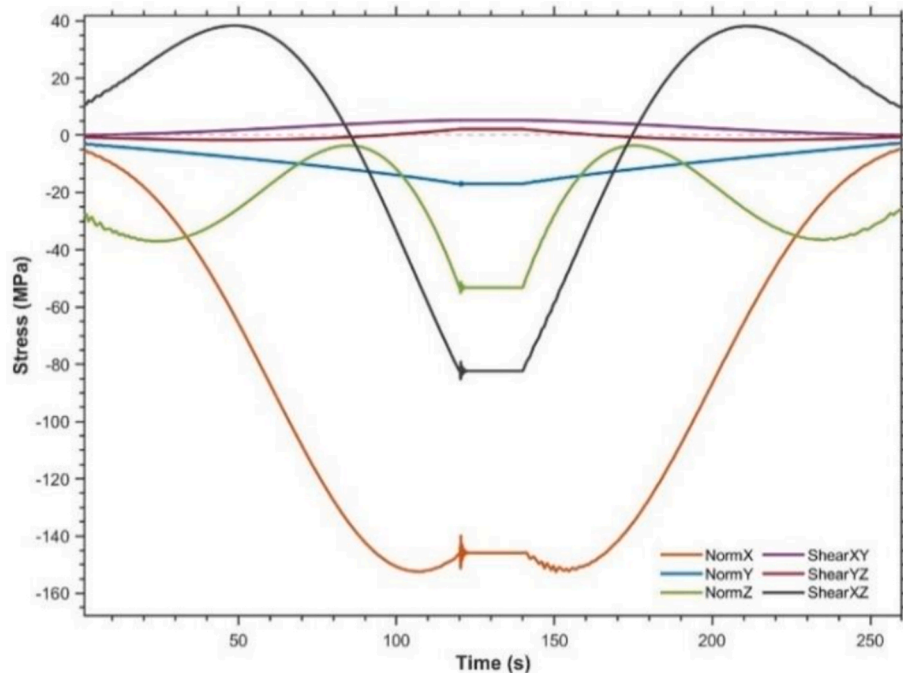


Fig. 17. Hot-spot load history.

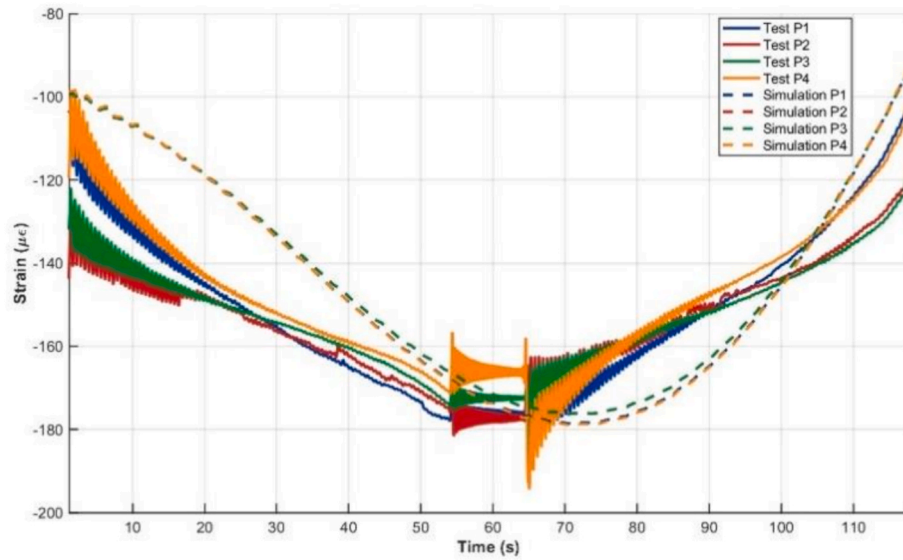


Fig. 19. Simulated and measured strain time histories.

Fig. 20 presents the damage-evolution curves $D_s(N)$, $D_v(N)$, and $D_{tot}(N)$. Both damage components exhibit nonlinear accelerated accumulation, and the volumetric damage at failure is approximately 8.8 times the deviatoric damage. As N approaches N_f , D_{tot} rapidly approaches the critical value D_c , indicating failure driven by the combined action of the two damage components. The volumetric/deviatoric decomposition framework continuously tracks the relative contributions of the two mechanisms throughout the life, providing quantitative, full-history diagnostic information on failure-mode evolution. This capability is not available in a single-scalar damage model.

To further assess predictive consistency, three classical multiaxial fatigue methods are considered for comparison: the S–N curve approach based on the von Mises equivalent stress amplitude with Miner's linear damage rule; the Findley critical-plane method, which searches all candidate planes for the maximum shear–normal coupled parameter; and the Brown–Miller method, a critical-plane criterion formulated in terms of a strain-based parameter. The predicted lives from the four methods are listed in Table 8.

The four predictions are of the same order of magnitude (9.76×10^4 – 1.43×10^5 cycles), and this consistency supports the overall plausibility of the predicted life level. Since a component-level fatigue failure benchmark is not available for this engineering case, the comparison in

Table 8 is intended to assess prediction consistency and mechanism interpretability rather than to establish a definitive ranking among the methods. All three classical methods predict shorter lives than the volumetric/deviatoric CDM (ratios of 0.68–0.83), indicating that the classical approaches are relatively more conservative. The Findley criterion, which emphasizes shear damage, yields the shortest predicted life. Combined with the damage-partitioning result ($r_v=89.8\%$), the present case is dominated by volumetric damage. Because the Findley criterion attributes all damage to a shear mechanism, it may overestimate the effective accumulation rate when volumetric damage actually dominates, thereby yielding conservative predictions. This comparison conversely corroborates the necessity of damage-component separation for failure-mechanism interpretation.

It should be emphasized that the classical methods output only a life estimate and cannot provide quantitative information on failure mechanisms. In contrast, the volumetric/deviatoric CDM provides life prediction together with the full evolution histories $D_s(N)$ and $D_v(N)$ and the final partitioning state (A -index, r_v), offering physical evidence beyond a single life number for mechanism-based structural integrity assessment. This failure-mode diagnostic capability is the central feature that distinguishes the volumetric/deviatoric decomposition framework from all comparison methods.

5. Conclusions

This study develops and validates a two-scale CDM model with separate deviatoric and volumetric damage variables for multiaxial fatigue life prediction. The conventional single scalar damage variable is replaced by independently evolving deviatoric and volumetric damage variables, enabling unified fatigue-life prediction, mechanism-resolved diagnostics, and complete damage-trajectory output. The main conclusions are summarized as follows:

- (1) A two-scale CDM model with separate deviatoric and volumetric damage variables D_s and D_v is established, driven by stress-state-

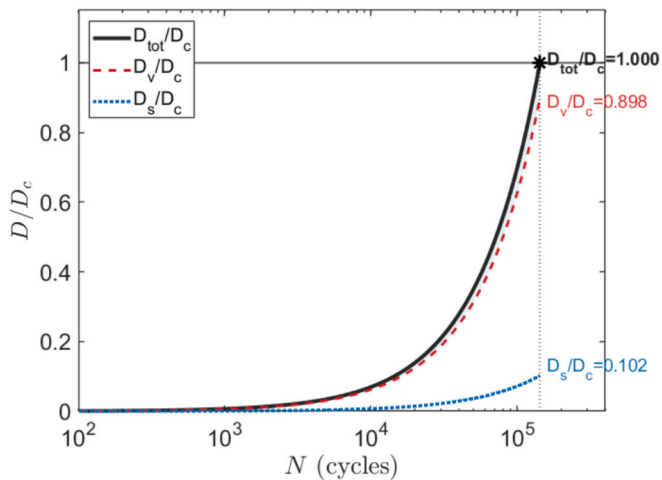


Fig. 20. Evolution of deviatoric and volumetric damage under the engineering loading.

Table 8

Comparison of fatigue-life predictions.

Method	Volumetric/Deviatoric CDM	SN/Miner	Findley	Brown–Miller
Life (cycles)	1.430×10^5	9.839×10^4	9.763×10^4	1.189×10^5

dependent damage strength functions $S_s(\eta^\mu, \xi^\mu)$ and $S_v(\eta^\mu, \xi^\mu)$. A controlled ablation study confirms that the stress-state-dependent $S(\eta, \xi)$ resolves the path-dependence problem of fixed-S formulations, while the deviatoric and volumetric separation further captures mixed-mode damage evolution, improving Path C accuracy from 66.7% to 88.9%.

- (2) Model parameters are calibrated from 41 fatigue data points of 7050-T7451 aluminum alloy under three loading paths, achieving $F_2 = 63.4\%$ and $F_4 = 90.2\%$. External validation on independent MIL-HDBK-5H S–N data without parameter refitting yields $F_4 = 100\%$ for both orientations, demonstrating robust parameter transferability.
- (3) The model provides mechanism-diagnostic outputs including damage trajectories $D_s(N)$ and $D_v(N)$, the volumetric damage fraction r_v , and the damage-partitioning A-index. The r_v varies continuously from 0% at $\eta = 0$ to 83.0% at $\eta = 1/3$, with a crossover at approximately $\eta = 0.167$ establishing a boundary between shear-dominated and void-growth-dominated regimes. Notably, $D_v = 0$ in pure torsion confirms the structural correctness of the deviatoric and volumetric separation.
- (4) The calibrated parameters are applied to fatigue assessment of a critical location in an aircraft cargo-door lifting mechanism under a full multiaxial stress-component history, predicting $N_f = 1.430 \times 10^5$ cycles with pronounced volumetric-damage dominance at failure ($r_v = 89.8\%$). Benchmarking against S–N/Miner, Findley, and Brown–Miller methods shows comparable life estimates, while the proposed model additionally provides complete $D_s(N)$ and $D_v(N)$ evolution histories and failure-mechanism partitioning that these classical approaches cannot provide.

Future work will be extended in the following directions. (1) incorporating non-proportional loading paths to quantify phase-difference effects on deviatoric/volumetric damage partitioning; (2) extending the model to variable-amplitude spectrum loading to investigate nonlinear cumulative interaction between the two damage variables; (3) integrating Bayesian inference to quantify uncertainty in calibrated parameters and to establish probabilistic life prediction procedures; and (4) applying the approach to additional critical aircraft components such as landing gears and control-surface actuation systems for standardized mechanism-resolved fatigue assessment.

CRedit authorship contribution statement

Yao-jia Han: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Chao-fan Li:** Writing – review & editing, Visualization, Software. **Cheng-wei Fei:** Writing – review & editing, Supervision, Resources, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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